Mathematical Modeling
Gravity-Mass Vehicles

When designing a solution to a problem it is often very useful to mathematically model different scenarios that your design will encounter. This way you can make changes to your design and predict how it will affect its performance.

You can do this modelling by hand or you can put it in a computer and have it do the work. You could write a computer program or use modelling programs like desmos, but you can also use a spreadsheet program like Google Sheets. This is what we will be using in this lesson.

Our goal today is to predict what our final velocity should be if all of the falling masses potential energy is converted to kinetic energy. We will also use mathematical modeling to determine the best way to design our vehicles for maximum final velocities

Please complete the next section and the first part of the following section before you test your vehicle

<u>Setting up Your Model</u>

Before we start using Google Sheets to model our gravity-mass cars we are going to collect some data to enter into our model and make sure we have our equations ready to enter into the spreadsheet.

Weight of Vehicle without falling mass:
Use one of the scales to measure the weight of your vehicle without the falling mass. Record it below. Make sure you record units as well. If you use a bathroom scale this will be in lbs. :

_____

Mass of falling mass:
Use one of the postal scales to measure the mass of your falling mass. Record it below. Make sure you record units as well (these should be in grams, please convert to kg):

_____

Height of Drop:
Measure how far the mass will drop as your car goes forward. This is the distance that the mass travels. Record this measurement in meters below:

_____

Potential Energy/Total Energy Available:
The gravitational potential energy of the falling mass represents the total energy of our gravity mass vehicle system before it starts falling. In order to predict the final velocity of the vehicle we need to know how much total energy is available. Write the equation for the gravitational potential energy below:

_____

Final Velocity Calculation:
Your vehicle reaches its greatest velocity (its final velocity) when the mass falls to its lowest point and its potential energy goes to zero and all the energy gravity-mass vehicle system has, theoretically, gone to kinetic energy. Using the concept of conservation of energy and the equations for gravitational potential energy and kinetic energy write a relationship between the initial energy and the final kinetic energy.

Using the relationship above solve for the final velocity to get an equation for the theoretical final velocity of your vehicle.

Building Your Model

You can use the Google sheet Gravity Mass Vehicle Modeling Template on my DP under Projects to work through this, or create your own. The instructions below assume you are using the template

1. Make a copy of the template or create your own sheet. These instructions assume you are using the template. You need one sheet per team. Share this sheet with Ms. Caroline and all your teammates.

2. Use the information you entered in the section above for "Weight of Vehicle without falling mass", "Mass of falling mass", and "Height of Drop" and enter it into the spreadsheet next to the corresponding titles

   Note: When you enter the weight of the vehicle put it under lbs and it will automatically change it to kg.

3. In the cell next to the "Actual Potential Energy/Total Available Energy" label (this is cell B11) you will be entering the equation to calculate the Gravitational Potential Energy of your falling mass. You should have written the equation in the previous section. In the spreadsheet enter "=B5*B7*B9".

   Note: You should notice that these values match the variables in the equation you wrote above
4. In the cell next to the "Predicted Final Velocity" label (this is cell B11) you will be entering the equation to calculate the theoretical final velocity of your vehicle. You should have written the equation in the previous section. In the spreadsheet enter "=sqrt(2*B11/(C3+B9))".
   Note: You should notice that these values match the variables in the equation you wrote above

This finishes your predictions for your vehicle. You can now test your pre-prototype at this point. If you want part of your team can continue to work on building your model as the other half tests your vehicle.

Continuing on with your model. In this section you will be testing to see how changing different aspects of your vehicle (mass and height of the falling mass) affects your final velocity. First we will examine what happens when you change the mass of the falling mass.

5. I have pre-set the mass range for you in column E. It goes from 50 g to 1 Kg If you would like to explore a different range of masses please change this column.

6. In cell F2 under the “Final Velocity” heading you will be entering the equation to calculate final velocity. You will be using the mass of your vehicle and the height of your vehicle as constants (variables that don’t change). In the spreadsheet in cell F2 enter “=sqrt(2*E2*B$5*B$7/(C$3+E2))” and push enter.

   Note: Recall that the “$” holds that cell constant as you drag the box down.

7. Click on cell F2 again and then click and hold on the small square in the bottom selection rectangle. Hold as you drag down column F until the column is populated. You have now calculated final velocity for all of these masses.

Next we will be looking at how the final velocity changes as we change the height that the mass falls. We will be holding all the other variables, including the mass of the falling mass, constant.

8. I have pre-set the height range for you in column H. It goes from 5 cm to 1 m If you would like to explore a different range of heights please change this column.
9. In cell I2 under the “Final Velocity” heading you will be entering the equation to calculate final velocity. You will be using the mass of your vehicle and the mass of your falling mass as constants (variables that don’t change). In the spreadsheet in cell I2 enter “=sqrt(2*B$9*B$5*H2/(C$3+B$9))” and push enter.

   Note: Recall that the “$” holds that cell constant as you drag the box down.

10. Click on cell H2 again and then click and hold on the small square in the bottom selection rectangle. Hold as you drag down column H until the column is populated. You have now calculated final velocity for all of these heights.

Reflecting on Your Results

Please answer the following questions in your notebook. You should also share your spreadsheet with Ms. Caroline and your partners and print out your data from the sheet to put in your notebook.

1. Compare the final velocity of your vehicle with your predicted final velocity. If the two velocities were not the same, what accounts for the differences between them? (In other words, where did the other energy go?)
2. Use your spreadsheet to create a graph of final velocity vs. mass (put this in your notebook). What happens as the mass of the falling mass increases?
3. Use your spreadsheet to create a graph of final velocity vs. height (put this in your notebook). What happens as the height that the mass falls increases?
4. Using the models and graphs you have developed that show how the height and mass of the falling mass affects velocity. How would you use of this mechanism in your sculpture to create the greatest final velocity?