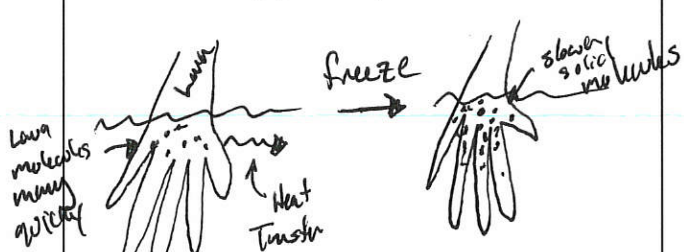
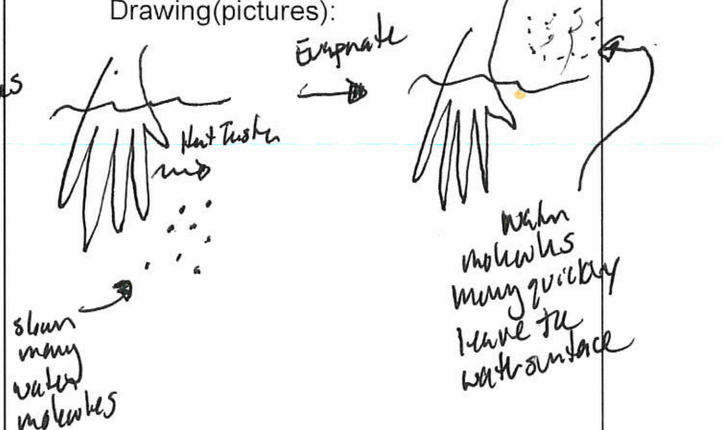
# 2019 Fall Semester Final Review

Name: Solutions Class:______

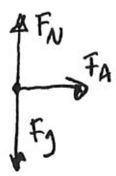
1. In the Disney Movie "Moana", the main character, Moana, tricks the lava monster, Te Ka, by using the fact that the lava monster is made of lava and that when lava meets the water, lava transfers heat energy rapidly to the sea water. Te Ka tries to grab Moana, and Moana, being too quick, slips away and Te Ka's lava hand plunges into the ocean, turning into solid rock. This happens multiple times until Moana is hidden in the fog that forms. In the boxes below indicate what is happening to TeKa's hand and the sea water using both words and pictures.

   If you would like, you can view a video of this scene by following this link or going to my DP

| Te Ka's Hand | Sea Water |
|---|---|
| Description(words): Heat Energy is transferred from Te Ka's Hand to the sea water. The Loss of Energy causes the molecules in Te Ka's hand to slow down and TeKa's hand to freeze into a solid. | Description(words): Heat Energy is transferred from Te Ka's Hand to the sea water. The sea water gains energy which cause the water molecules to move faster. The molecules Heat up (speed up) so much that they evaporate, forming the fog |
| Drawing(pictures):   | Drawing(pictures):   |

2. A student is travelling west on San Marcos Blvd in their car which has a mass of 1500 kg. The student begins their journey stopped at the intersection of Rancho Santa Fe and San Marcos Blvd. When the light turns green the car accelerates for 600 m with an acceleration of 4 m/s² until it reaches High Tech High North County. The student then sees Ms. M and Ms. Abdullah outside the school and immediately stops accelerating.

   a. What speed was the car travelling when they passed Ms. M and Ms. Abdullah?




$\underline{X}$

$a = 4 m/s^2$   $t_o = 0 s$

$V_o = 0 m/s$   $t_f = ?$

$V_f = ?$   $m = 1500 kg$

$S_o = 0 m$   $F = ?$

$S_f = 600 m$

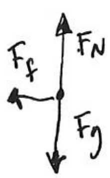
$V_f^2 = V_o^2 + 2a\Delta S$

$V_f^2 = 0^2 + 2 \cdot 4 \cdot (600 - 0)$

$\sqrt{V_f^2} = 0 + 8 \cdot 600 = \sqrt{4,800}$

$\boxed{V_f = 69.3 m/s}$

   b. If the student didn't accelerate again and the kinetic coefficient of friction of their tires with the road is $\mu_k = 0.7$, how fast was the car going when it reached the red stop-light at South Pacific Street, 300 m away?




$\underline{X}$

$a = ?$   $F_f = ?$

$V_o = 69.3 m/s$   $S_o = 0 m$

$V_f = ?$   $S_f = 300 m$

$m = 1500 kg$   $t_o = 0$

$t_f = ?$

$F_f = \mu_k \cdot F_N$

$F_N = mg$

$F_f = -0.7 \cdot 1500 \cdot 9.8 = -10,290 N$

$F_f = ma = -\frac{1500 \cdot a}{1500} = \frac{-10,290}{1500} \rightarrow a = -6.86 m/s^2$

$V_f^2 = V_o^2 + 2a\Delta S$

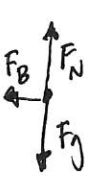
$V_f^2 = 69.3^2 + 2 \cdot (-6.86) \cdot 300$

$V_f^2 = 686.49$

$\sqrt{V_f^2} = \sqrt{686.49}$

$\boxed{V_f = 26.2 m/s}$

   c. If the student had 5 seconds to brake before reaching the light. What force did the car's brakes need to apply in order to stop the car safely at the red light?




$\underline{X}$

$a = ?$

$V_o = 26.2 m/s$

$V_f = 0 m/s$

$S_o = 0 m$

$S_f = ?$

$t_o = 0 s$

$t_f = 5 s$

$m = 1500 kg$

$F_B = ?$

$a = \frac{V_f - V_o}{t_f - t_o} = \frac{0 - 26.2}{5 - 0} = -\frac{26.2}{5}$

$= -5.24 m/s^2$

$\boxed{F_B = ma = 1500 \cdot (-5.24) = -7,860 N}$

3. The Following Questions refer to the motion above

    a. Draw a Position vs Time, Velocity vs time, and Acceleration vs. time graph for the motion of the car in question #2. You will need to determine the timing of the action and how far the car travelled while braking in order to draw each graph accurately.

#1 Car accelerates - need to find $t_f$

$s(t_f) = s_0 + V_0 t_f + \frac{1}{2} a t_f^2$

$600 = 0 + 0 \cdot t_f + \frac{1}{2} \cdot 4 \cdot t_f^2$

$\frac{600}{2} = \frac{2t_f^2}{2}$

$\sqrt{300} = \sqrt{t_f^2}$

$17.3s = t_{f_1}$

#2 Car decelerates - find $t_f$

$300 = 0 + 69.3t_f - \frac{1}{2} \cdot 6.86 t_f^2$

$300 = 69.3t_f - 3.43t_f^2$

$t_f = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$ (only + makes sense)

$a = 3.43,\ b = 69.3,\ c = -300$

$t_f = \frac{-69.3 + \sqrt{69.3^2 - (4 \cdot 3.43 \cdot -300)}}{2 \cdot 3.43}$

$t_{f_2} = \frac{-69.3 + \sqrt{8918.5}}{6.86} = 3.7s$

#3 Car Brakes and decelerates find distance

$s(t=5) = 0 + 26.2 \cdot 5 - \frac{1}{2} \cdot 5.24 \cdot 5^2$

$s_f = 131 - 65.5 = 65.5m$

$t_0 = 0s$

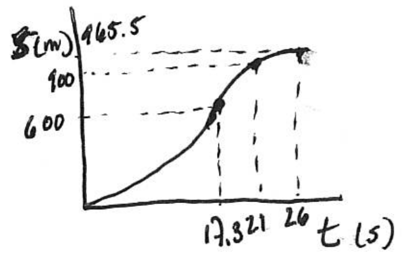
$t_{f_1} = 17.3s$

$t_{f_2} = 17.3 + 3.7 = 21s$

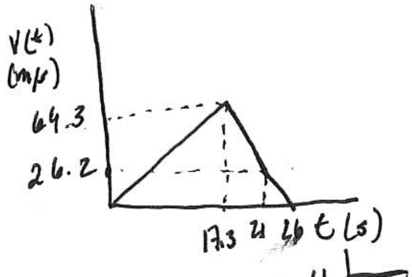
$t_{f_3} = 21 + 5 = 26s$

    b. Write the piecewise function that describes the above position vs time, velocity vs time, and acceleration vs. time graphs.

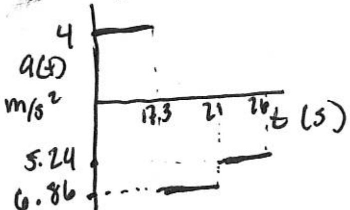
$$s(t) \begin{cases} 0 \le t < 17.3 & 2t^2 \\ 17.3 \le t < 21 & 600 + 69.3(t - 17.3) - 3.43(t-17.3)^2 \\ 21 \le t \le 26 & 900 + 26.2(t-21) - 2.62(t-21)^2 \end{cases}$$

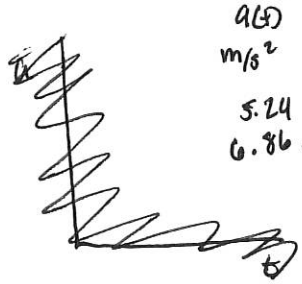



$$v(t) \begin{cases} 0 \le t < 17.3 & 4t \\ 17.3 \le t < 21 & 69.3 - 6.86(t - 17.3) \\ 21 \le t \le 26 & 26.2 - 5.24(t - 21) \end{cases}$$



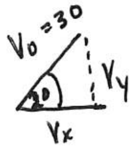

$$a(t) \begin{cases} 0 \le t < 17.3 & 4 \\ 17.3 \le t < 21 & -6.86 \\ 21 \le t \le 26 & -5.24 \end{cases}$$



4. Connor fires an arrow with an initial velocity of 30 m/s at an angle of 20 degrees.

   a. What is the vertical velocity of the arrow?




$$\frac{V_y}{V_0} = \sin(20) = \frac{V_y}{30} \rightarrow V_y = 30\sin(20) = \boxed{10.3 \text{ m/s}}$$

   b. What is the max height that the arrow achieves?

$$V_{max} = 0 \qquad 0 = 10.3 - 9.8\, t_{max}^2 \rightarrow \frac{-10.3}{-9.8} = \frac{-9.8\, t_{max}}{-9.8}$$

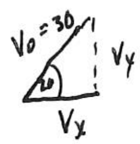
$$t_{max} = 1.05\text{s}$$

$$s_y(t_{max}) = s_{oy} + V_{oy}t_{max} + \tfrac{1}{2}a_y t_{max}^2 = 0 + 10.3 \cdot 1.05 - \tfrac{1}{2} \cdot 9.8 \cdot 1.05^2 = \boxed{5.4\text{m}}$$

   c. How long does it take the arrow to reach that max height?

$$\boxed{t_{max} = 1.05\text{s}} \text{ (see above)}$$

   d. What is the horizontal velocity of the arrow?



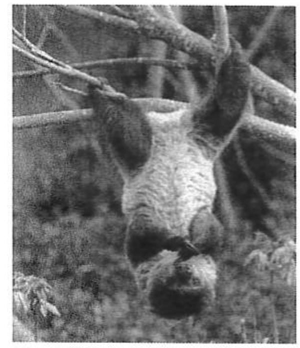

$$V_x = V_0 \cos(20) = 30\cos(20) = \boxed{28.2 \text{ m/s}}$$

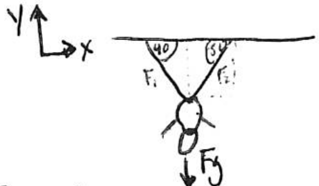
   e. How far does the arrow go before it hits the ground?

$$s_x(t_f) = s_{ox} + V_{ox}t_f = 0 + 28.2 \cdot t_f = 28.2 \cdot 2.1 = 59.2 \text{ m}$$

$$t_f = 2 \times t_{max} = 2 \cdot 1.05 = \boxed{2.1\text{s}}$$

5. A sloth, like the two toed one picture here, spends most of their life hanging. Imagine a sloth weighs 7 kg and is hanging upside down by two of their feet (like the one pictured here). If the sloths legs make angles of 40° and 50° with the horizontal, how much force must each leg contribute to keep the sloth from falling? (Be sure to include a drawing of the scenario and a Free Body Diagram)



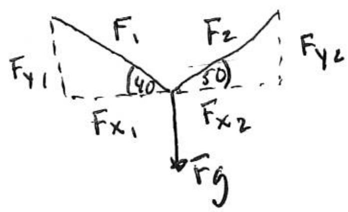


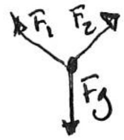

$F_1$ = Force in leg 1
$F_2$ = Force in leg 2

$F_{net} = 0$

$\sum F_x = F_{x_1} + F_{x_2} = 0$

$\sum F_y = F_{y_1} + F_{y_2} + F_g = 0$




$\frac{F_{x_1}}{F_1} = \cos(40) \rightarrow F_{x_1} = -F_1 \cos(40)$

$\frac{F_{x_2}}{F_2} = \cos(50) \rightarrow F_{x_2} = F_2 \cos(50)$

$\sum F_x = F_2 \cos(50) - F_1 \cos(40) = 0$

$F_2 \cos(50) = F_1 \cos(40)$

$F_2 = F_1 \frac{\cos(40)}{\cos(50)} = \frac{0.77}{0.64} F_1 = 1.2 F_1$

$\frac{F_{y_1}}{F_1} = \sin(40) \rightarrow F_{y_1} = F_1 \sin(40)$

$\frac{F_{y_2}}{F_2} = \sin(50) \rightarrow F_{y_2} = F_2 \sin(50)$

$F_g = -mg = -9.8 \cdot 7 = -68.6\,N$

$\sum F_y = F_1 \sin(40) + F_2 \sin(50) - 68.6N = F_1 \cdot 0.64 + 1.2F_1 \cdot 0.77 - 68.6$

$= 1.6F_1 - 68.6 = 0 \rightarrow \frac{1.6F_1}{1.6} = \frac{68.6}{1.6} \rightarrow \boxed{F_1 = 42.9\,N}$

$F_2 = 1.2 \times 42.9 = \boxed{51.48N}$

6. A sled is at the top of an icy hill that is 30m high and has an angle of incline of 25 degrees. The sled has a mass of 30 kg.

   a. What is the potential energy of the sled at the top of the hill?

$m = 30\,kg$
$g = 9.8\,m/s^2$
$h = 30\,m$
$\theta = 25^\circ$

$$GPE = mgh = 30 \cdot 30 \cdot 9.8 = \boxed{8,820\,J}$$

   b. What is the velocity of the sled at the bottom of the hill?

$$GPE = KE = \frac{1}{2}mv^2 = 8,820 = \frac{1}{2} \cdot 30 \cdot v^2$$

$$\frac{8,820}{15} = v^2$$

$$\sqrt{588} = \sqrt{v^2}$$

$$\boxed{24.2\,m/s = v}$$

   c. If Zachary, who has a mass of 15 kg, was sitting on the sled what would the velocity of the sled be at the bottom of the hill?

new mass $= 30 + 15 = 45\,kg$

$$GPE = mgh = 45 \cdot 30 \cdot 9.8 = 13,230\,J$$

$$GPE = KE = \frac{1}{2}mv^2 \rightarrow 13,230 = \frac{1}{2} \cdot 45 \cdot v^2$$

$$\frac{13,230}{22.5} = 588 = v^2$$

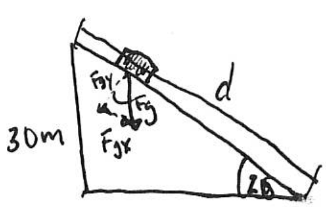
$$\boxed{v = 24.2\,m/s}$$

Same, mass doesn't matter.

   d. If the sled, with Zachary on it, was instead on mud which has a coefficient of kinetic friction of 0.2, what would the velocity of the sled be at the bottom of the hill?

Enrgy Loss to Friction = Work done by frictional force = $F_f \cdot$ distance

d = distance $\Rightarrow \sin(25) = \frac{30}{d} \rightarrow d = \frac{30}{\sin(25)} = 71\,m$




$F_f = F_N \cdot \mu_k$ $\quad \mu_k = 0.2$ $\quad F_N = F_g \cdot \cos(25) = mg\cos(25)$

$= 45 \cdot 9.8 \cdot \cos(25) =$

$F_f = 400 \cdot 0.2 = 80\,N$ $\quad = 400\,N$

Enrgy Loss $= F_f \cdot d = 80 \cdot 71 = 5,680\,J$

Subtract Enrgy Loss from GPE $\rightarrow 13,230 - 5,680 = 7,600\,J = \frac{1}{2}mv^2 = \frac{1}{2} \cdot 45\,v^2$

$$\frac{7,600}{22.5} = \frac{22.5\,v^2}{22.5} \rightarrow \sqrt{337.8} = \sqrt{v^2} \rightarrow \boxed{v = 18.4\,m/s}$$

7. A drainage pipe with an inner diameter of 2m, an outer diameter of 2.5 m, and a mass of 125 kg broke free from its transport truck 15 meters up a hill and is rolling down the street.

   a. Assuming it starts from a stop, what is the total energy of the pipe before it starts rolling down the hill?

$$GPE = mgh = 125 \cdot 9.8 \cdot 15 = \boxed{18,375\ J}$$

$m = 125\ kg \quad h = 15m$

$g = 9.8$

   b. What is the tangential velocity of the pipe when it reaches the bottom of the hill?

$$GPE = 18,375 = KE_{Trans} + KE_{rot} = \frac{1}{2}mv^2 + \frac{1}{2}I\omega^2 \qquad GPE = K_{Tot} = KE_{Trans} + KE_{rot}$$

$$\omega = \frac{v}{r} \rightarrow 18,375 = \frac{1}{2}mv^2 + \frac{1}{2}I\left(\frac{v}{r}\right)^2$$

$$I = \frac{1}{2}m(r_1^2 + r_2^2) = \frac{1}{2} \cdot 125(2^2 + 2.5^2) = 640.6$$

$$18,375 = \frac{1}{2} \cdot 125v^2 + \frac{1}{2} \cdot 640.6\frac{v^2}{2.5^2} = 62.5v^2 + 51.2v^2$$

$$\sqrt{v^2} = \sqrt{113.7} \qquad \boxed{v = 10.7\ m/s}$$

   c. What is the angular velocity of the pipe?

$$\omega = \frac{v}{r} = \frac{10.7}{2.5} = \boxed{4.3\ rad/s}$$

   d. If by some strange coincidence a solid metal column that had a radius of 2.5m fell off another truck at the same time and same place as the drainage pipe, would the drainage pipe or the solid metal column reach the bottom of the hill first? Why?

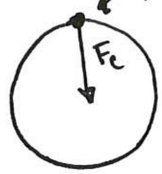
The solid metal Pipe would reach the bottom first Because its moment of Inertia (I) is smaller. This means less of the Total Energy is spent on Rotating the Pipe and more contributes to its total translational velocity

8. You are on a merry go round that has a radius of 3m. You notice that you complete a revolution every 2 minutes.
   a. What is your tangential velocity as you spin on the merry-go-round?

$$\omega = \frac{2\pi \cdot 1\,rev}{2 \cdot 60s} = 0.05\ rad/s \qquad \omega = \frac{v}{r} \Rightarrow 0.05 = \frac{v}{3}$$

$$\boxed{v = 0.16\ m/s}$$

   b. Assuming that you are holding onto the merry-go-round, what are the forces acting on you as you are spinning on this merry-go-round? Draw them a diagram depicting them below. Make sure to label each force and indicate its direction




Normal Force pointing out of the paper, Gravitational Force Pointing into the paper

   c. Assuming you weigh 75 kg. What are the magnitudes of the forces that you experience?

$$F_c = ma_c = m\frac{v^2}{r} = 75\,\frac{0.16^2}{3} = 0.64N$$

$$F_N = -F_g = 75 \cdot 9.8 = 735N$$

   d. Suppose you move in 1.5m towards the center of the merry-go-round. What is the magnitude of the force now?

$$F_c = m\frac{v^2}{r} = 75\,\frac{0.16^2}{1.5} = \boxed{1.28N}$$

9. A child's toy that is made to shoot ping pong balls consists of a tube, a spring (k = 18 N/m) and a catch for the spring that can be released to shoot the balls. When a ball is loaded into the tube, it compresses the spring 9.5 cm. (For the following questions ignore air resistance)
   a. Once released what force does the spring exert on the ping pong ball?

$F = Kx$ $\quad K = 18 \text{ N/m} \quad x = 9.5\text{cm} = 0.095\text{m}$

$F = 18 \cdot 0.095 = \boxed{1.71\text{N}}$

   b. If you shoot a ping pong ball straight up out of this toy, how high will it go?

I needed to give you a mass here. A ping pong ball has a mass of 2.7 grams.

$m = 2.7\text{g} = 0.0027 \text{ kg}$

$EPE = \frac{1}{2}Kx^2 = GPE = mgh$ ← hight

$\frac{1}{2} \cdot 18 (0.095)^2 = 0.0027 \cdot 9.8 \cdot h$

$0.08 = 0.026h \qquad h = \frac{0.026}{0.08} = \boxed{0.3\text{m}}$

   c. If you hold your hand out exactly where you shot the ping pong ball fromHow long will it take for the ping pong ball to fall back to your hand?

To reach max height.... it will take $\frac{1}{2}$ the time

$S(t_{max}) = S_0 + V_0 t + \frac{1}{2}at^2$

need to find $V_0$.... $KE = \frac{1}{2}mv^2 = \frac{1}{2}Kx^2 = EPE$

this is $V_0$!

Y

$a = -9.8$

$V_0 = ?$

$V_f = ?$

$S_0 = 0\text{m}$

$S_f = 0\text{ m}$

$V_{max} = 0 \text{ m/s}$

$S_{max} = 0.3$

$t_0 = 0$

$t_f = ? = 2t_{max}$

$t_{max} = ?$

$\frac{1}{2} \cdot 0.0027 \cdot V_0^2 = \frac{1}{2} \cdot 18 \cdot (0.095)^2$

$\frac{0.00135 V_0^2}{0.00135} = \frac{0.08}{0.00135}$

$\sqrt{V_0^2} = \sqrt{59.3}$

$V_0 = 7.7 \text{ m/s}$ ← Plug into displacement equation

$0.3 = 0 + 7.7t_{max} - \frac{1}{2}9.8t_{max}^2$

or you could do

$V_{max} = V_0 + at_{max}$

$0 = 7.7 - 9.8t_{max}$

$7.7 = 9.8t_{max} \rightarrow t_{max} = \frac{7.7}{9.8} = 0.8\text{s}$

$\boxed{t_f = 2 \cdot t_{max} = 1.6\text{s}}$